

Design of a pedestrian bridge using Warren Truss model

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Abstract

A recent investigation about the pedestrian safety outside Libertad High School shows a problem with the traffic, because when the students and old people try to cross the street, put their life at risk. So, to improve people's life and to avoid pedestrian accidents, a pedestrian bridge will be designed as an integrative Project, using Warren Truss model. In calculus, integer-differentials equations will be applied; in physics, the sum of forces will be calculated; the area and volume of some structural parts will be found in vector geometry, and finally it will be used AutoCAD software to design the pedestrian bridge in 3D.

Key words: pedestrian accidents, pedestrian bridge, pedestrian safety.

1. Introduction

Civil Engineering helps to design and to build several types of infrastructures, such as, airports, roads and bridges; these last ones are usually installed in places prone to accidents, which means that their objective is to allow the passage over lands with depressions, rivers and even seas. Their size and design can be diverse depending on the height, the load for which they are made, the length that they will cross and the place where they will be located. The materials can also change, depending on the environmental conditions to which they will be submitted. Warren Truss is a model patented in 1848 by English Engineer James Warren and consists of longitudinal bars joined only by transverse bars, forming triangles along their entire length, ensuring that no beam is subject to bending or buckling, but only to tension or compression. The loads in the diagonals alternate between compression and tension, while the elements near the center support tension and compression in response to live loads. This configuration combines great rigidity with economy of materials and therefore can be relatively light, because the beams have the same length.

The next project describe the situation of the community near Libertad High School. When the people, especially students, want to cross the street, they put themselves at risk of having an accident. Taking into account the previous ideas, a pedestrian bridge will be the best solution for the problem. The construction

material will be steel because this material will provide safety and resistance to the pedestrian bridge.

Objectives

General

To design a pedestrian bridge using Warren Truss model in order to benefit the community close to Libertad High School, because the vehicular congestion in the street located in front of it, could cause pedestrian accidents. This project will be carried out applying the knowledge acquired in each of the courses of period 2018A.

Specific Objectives

- To find the maximum points, minimum points, growth points, degrowth points using derivatives equations and to calculate the area under the curve using integral equations.
- To calculate the sum of forces and the live and dead loads in the pedestrian bridge.
- To calculate the area of the steel sheet and the volume of the columns from the structure.
- To use AutoCAD Software Application for the graphic design of the pedestrian bridge.

Description of the Problem

People and students who usually take the route outside Libertad High School, especially in the rush hour when they want to go to their homes, work or school, face a big problem, because being this a main street, the traffic is heavier than other streets due to the presence of cars, motorcycles and trucks. Taking into account that when the students and other people try to cross the street put their life at risk, this project intends to give a solution with the building of a pedestrian bridge, using Warren Truss model which would ensure people's lives. The construction will also be carried out with the purpose of being used by those people who do not own a private vehicle or who do not have the monetary resources to pay for public transportation.

For this reason, they have to get to their homes and to their works on foot.

Methodology

The methodology of this project incorporates the knowledge and skills acquired along the courses assigned for 2018A period: Calculus, Physics, Vector Geometry and AutoCAD. Firstly, it is necessary to perform derivatives equations to find the critical points, maximum points, minimum points, the inflection points and the change of concavity. Integral equations will also be used to calculate the area under the curve. Secondly, the Newton's second law will be applied for calculating the sum of the forces and how they act in the pedestrian bridge; besides, live, dead and total loads will be analyzed, and how these ones affect the construction of the bridge. Thirdly, the area of surface of the bridge, the steel sheet and the total volume of the columns will be gotten. This process will be made with the cross product and dot product. It is worth highlighting that the design of bridge was worked in a tool called AutoCAD.

The structure will be made in 2D just employing lines and circles, with the objective of getting the top, front and lateral view. Then the design will be changed because it is required in 3D and to get this result the structure will be made again, but this time it is going to use elements like cylinders, cubes and other type of figures in three dimensions. The measures will also be specified by delimiting the distances.

Discussion

Calculus

Table 1. Bridge dimensions

Dimension	Measure
Width	2 m
Length	16 m
Height	11 m

To determine the equation of the curve, it took into account first that the function of the parable is:

$$f(x) = x^2 \quad (1)$$

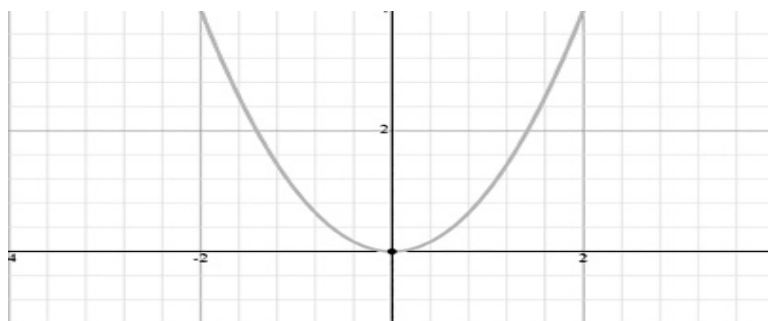


Figure 1. Function of the parable

To invert the equation 2, a negative sign must be added in the next way:

$$f(x) = -x^2 \quad (2)$$

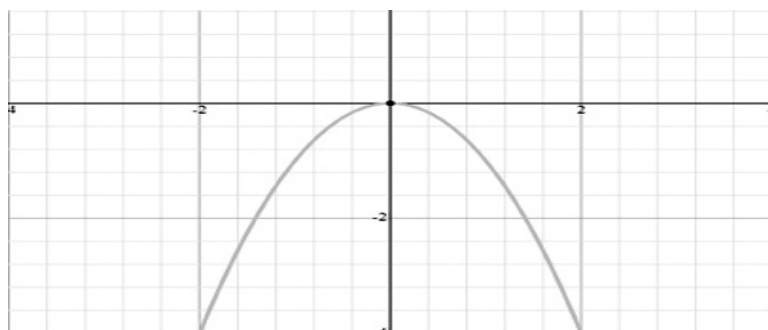


Figure 2. Second parable

To prolong the length in the x-axis, the equation 3 must be divided between 32 and, finally, to determine the height 2 units are added.

$$f(x) = \frac{-x^2}{32} + 2 \quad (3)$$

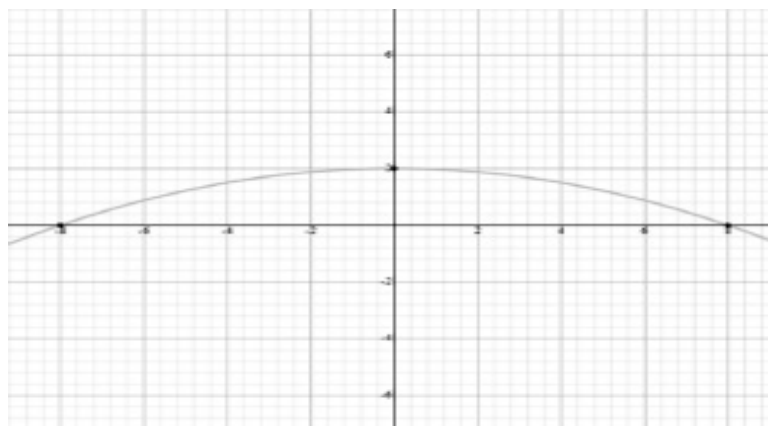


Figure 3. Curve function.

First and second derivatives application

First derivative criterion: To find the critical points, maximum points, minimum points, growth points and degrowth points.

- **Critical points:** in order to obtain the critical points of the function of the curve, the equation 3 will be first derivative.

$$f(x)' = \frac{-x}{16} \quad (4)$$

Equation 4 is equal to zero and the variable x will be cleared

$$\frac{-x}{16} = 0 \quad (5)$$

$$x = 0$$

Finally the value obtained from the variable X is replaced in Equation 3, thus producing the critical point on the Y axis.

$$f(0) = \frac{(-0)^2}{32} + 2 \quad (6)$$

$$f(0) = 2$$

- **Growth and degrowth points:** To obtain them, the value of x is taken as the reference point; negative and positive values are chosen and replaced in Equation 4.

$$f(-2) = \frac{1}{8} \quad (7)$$

$$f(1) = \frac{-1}{16} \quad (8)$$

Second derivative criterion: To find the inflection points and the change of concavity

- **Inflection points and concavity:** To apply second derivative to equation 4

$$f(x)'' = \frac{-1}{16} \quad (9)$$

Area under the curve

To find the area under the curve the following equation will be considered:

$$A = \int_b^a [f(x) - g(x)] dx \quad (10)$$

Values of A and B equivalent to the integration limits, for this case are 8 and -8 respectively, the value of f (x) indicated by the equation 3 and g (x) which is worth zero

$$A = \int_{-8}^8 \left(\frac{-x^2}{32} + 2 \right) dx \quad (11)$$

The result of equation 12 will be obtained by the previous integral.

$$A = \frac{-x^3}{96} + 2x \quad (12)$$

The integration limits will be evaluated replacing x with the values of 8 and -8 as shown in equation 13

$$A = - \left(\frac{8^3}{96} - \frac{(-8)^3}{96} \right) + [2(8) - 2(-8)] \quad (13)$$

The final result is:

$$A = 21.33m^2 \quad (14)$$

Physics

Dead load

To find the dead load, it will be first calculated the mass of the cover that is made of steel. This will be carried out through equation 15:

$$m = d \cdot v \quad (15)$$

The density of the material is equal to 7850 kg/m³. To calculate the volume it will be considered that the cover is a hollow semi cylinder, therefore equation 16 is applied:

$$V = \frac{\pi}{8} h (D^2 - d^2) \quad (16)$$

D is the value of the width of the cover; d, the width of the opening in the center and h, the length. The Table 2 presents the values of each variable.

Table 2. Cover dimensions

Variable	Equivalence
D	2 m
d	1.8 m
h	16 m

Table values are replaced in equation 16 and resolved for finding the volume

$$V = \frac{\pi}{8} 16m[(2m)^2 - (1.8m)^2]$$

$$V = 4,78m^3$$

The volume found and the density must be replaced in equation 15 to find the mass.

$$m = 7850kg/m^3 \cdot 4,78m^3$$

$$m_c = 37.52kg$$

Then the mass of the vertical supports will be calculated; for this first it must be found the volume replacing the values in Table 3 in the equation 17

$$V = \pi hr^2 \quad (17)$$

Table 3. *Dimensions of the vertical supports*

Variable	Equivalence
h	5 m
r	0.035 m

It is considered that these supports are of the same material as the cover, therefore it is used the same density value.

$$V = \pi \cdot 5m \cdot (0.035m)^2$$

$$V = 0.019m^3$$

The volume obtained along with the density is replaced in equation 15 to obtain the mass.

$$m = 7850kg/m^3 \cdot 0.019m^3$$

$$m = 149.15kg$$

Since the total of supports is equal to 8, the mass found is multiplied by this number.

$$m_{sv} = 1193.2kg$$

For the floor slab a type of steel sheet that has a mass of \$95.47 kg \$ is used. In total they are 19 laminates; therefore:

$$m_l = 1813.93kg$$

Now for finding the total mass, the value of the cover, the supports and the sheet are added with the aid of the equation 18

$$m_{total} = m_c + m_{sv} + m_l \quad (18)$$

$$m_{total} = 3044.65kg$$

Finally, to calculate the weight force exerted by the total mass, the equation 19 is used.

$$F_w = mg \quad (19)$$

The value of the total mass and of gravity are replaced.

$$F_w = 3044.65kg \cdot 9.87m/s^2$$

$$F_w = 30050.7N$$

Live load

According to the design criteria and parameters for pedestrian bridges, the live load corresponds to 450 kg/m², so, taking into account the above, the following process must be done:

$$m_l = 450Kg/m^2 \cdot 32m^2$$

$$14400Kg$$

The previous result corresponds to the mass that the bridge plate supports; this must be replaced in the following equation:

$$F_l = mg$$

$$F_l = 14400 \cdot 9.87m/s^2$$

$$F_l = 142128N$$

Total load

To calculate the total load you must add the live load and the dead load.

$$Totalload = 172133.7N \quad (20)$$

$$Totalload = 142128N + 30050.7N$$

Nodes

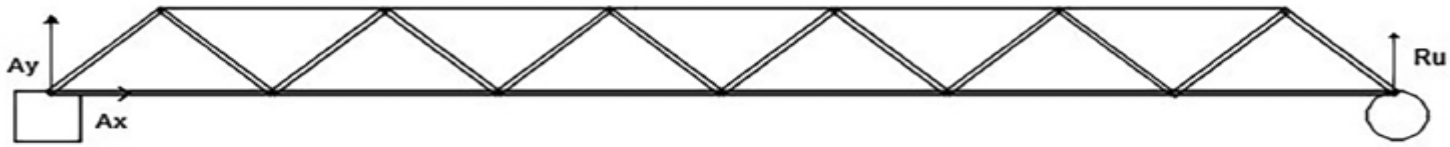


Figure 4. Truss

The bridge will have two trusses, one on each side, holding the slab; to get the value of its weight applies equation 19, knowing that their mass is 1813.93 kg.

$$F_w = 1813.93kg \cdot 9.87m/s^2$$

$$F_w = 17903.48N$$

The result obtained must be divided by the number of trusses

$$\frac{17903.48N}{2} = 8951.74N$$

Then, divide the value found between the numbers of upper nodes that carries the truss, which for this case are 10; therefore

$$\frac{8951.74N}{10} = 895.174N$$

Since both trusses are symmetrical, the results of one will also serve for the other.

For the truss are placed a fixed support on the left side and a mobile to the right. The sum of torques was carried out taking the moment in the fixed support. This process took place through the following equation:

The reaction in the U-node must be replaced in equation 22 corresponding to the sum of forces in the y-axis of the same node

$$\sum \tau_A = 0 \quad (21)$$

$$\tau_w + R_U = 0$$

$$-8951.74N(9.57m) + R_U(19.14m) = 0$$

$$R_U = \frac{8951.74N(9.57m)}{(19.14m)}$$

$$R_U = 4475.87N$$

$$\sum F_{Uy} = 0 \quad (22)$$

$$R_U(\text{Sen}45) + F_{UT} = 0$$

$$F_{UT} = \frac{-4475.87(\text{Sen}45)}{\text{Sen}45}$$

$$F_{UT} = -4475.87$$

The UT force is replaced in equation 23 to find the US force

$$\sum F_{Ux} = 0 \quad (23)$$

$$-F_{US} - F_{UT}(\cos 45) = 0$$

$$F_{US} = -(-4475.85N)(\cos 45)$$

$$F_{US} = 3164.90N$$

The rest of the forces are recorded in the results section

Vector Geometry

Areas and Volumes

- **Areas:** The bridge floor area is calculated through the cross product with the following points:

$$U(8, 0, 2) \quad V(8, 0, 0) \quad W(-8, 0, 2)$$

First of all it must be taken out a vector between UV and UW

$$\overrightarrow{UV} = \vec{A} = 2\vec{k}$$

$$\overrightarrow{UW} = \vec{B} = 16\vec{i}$$

With these results it is known that the vectors **i** and **j** for Vector **A** and vectors **j** and **k** for vector **B** are equal to **0**. Therefore:

$$\vec{A} \times \vec{B} = 0\vec{i} + 32\vec{j} - 0\vec{k}$$

$$|\vec{A} \times \vec{B}| = \sqrt{32^2}$$

$$|\vec{A} \times \vec{B}| = 32$$

The total area of the floor is 32 m²

- **Volumes:** the volume of one of the columns is calculated through the cross product with the following points:

$$P(8, -1, -0.5) \quad Q(8, -1, 0)$$

$$R(7.5, -1, -0.5) \quad S(8, -6, -0.5)$$

Three vectors are obtained between PQ, PR and PS

$$\overrightarrow{PQ} = \vec{A} = 0.5\vec{k}$$

$$\overrightarrow{PR} = \vec{B} = -0.5\vec{i}$$

$$\overrightarrow{PS} = \vec{C} = -5\vec{j}$$

Cross product is carried out between vectors **A** and **B** and finally multiplies that result by vector **C**

$$\vec{A} \times \vec{B} = 0\vec{i} - 0.25\vec{j} - 0\vec{k}$$

$$|\vec{A} \times \vec{B}| \cdot \vec{C} = (-0.25\vec{j})(-5\vec{j})$$

$$|\vec{A} \times \vec{B}| \cdot \vec{C} = 1.25$$

The volume of a column is 1.25 m³. So the total volume of the 6 columns is 7.5 m³.

Results

To the part of calculus with the results obtained with the equation 6 it can be concluded that the critical point of the function is P1 (0, 2); this in turn could be a maximum or a minimum, because if in the growth and degrowth parts the curve goes from growing to decrease, is a maximum, but if the opposite happens, it's a minimum. Taking into account the results of the equations 7 and 8 it can be observed that when taking the value of -2 the result was positive, which means that the curve grows in the interval $(-\infty, 0)$. On the other hand replacing the value 1 the result was negative; then from the interval $(0, \infty)$ the curve decreases. It is therefore concluded that the critical point is also a maximum point. According to the result of equation 9 it is established that there are no inflection points or change of concavity at any point because it is a constant. Finally, the result in the equation 14 refers the approximate area value that will take the curve of the bridge deck.

The dead loads obtained through the respective equations determine only the structural elements that make up the pedestrian bridge. However, the living burdens represent the weight of the pedestrian congestion; that is to say of the people who will pass by the bridge. One must bear in mind that this is expressed in a maximum way in order to determine whether the structure will be able to withstand the weight with traffic. Also, the total sum represents the maximum load for which this pedestrian bridge will be designed. Finally, Table 4 shows the results obtained when applying the summation of forces in node A. This represents which bars are compression and which to tension.

Table 4. Forces

Fuerza	Tensión	Compresión
AB	-	-12071.57N
AC	-	-
R _A	12071.57N	-
BC	10805.61N	-
BD	56777.03N	-
CE	-	-49968.19N
CD	-	-10805.61N
DF	42390.76N	-
DE	9539.65N	-
EF	-	-9539.65N
EG	-	-36477.09N
FH	29794.83N	-
GI	-	-24776.33N
GF	8273.70N	-
GH	-	-8273.70N
HI	7007.74N	-
HJ	18989.22N	-
IK	-	-14034.03
IJ	-	-7007.74N
JL	9973.95N	-
JK	5741.78	-
KM	-	-5913.93N
KL	-	-5741.78N
LN	2749.04N	-
LM	4475.82N	-
MO	415.83N	-
MN	-	-4475.82N
NP	-	-2685.55N
NO	3209.86N	-
OQ	4955.25N	-
OP	-	-3209.86N
PR	-	-6329.8
PQ	1943.90N	-
QS	7704.33N	-
QR	-	-1943.90N
RT	-	-5434.62N
RS	-	-3209.87N
SU	3164.90N	-
ST	3209.87N	-
TU	-	-4475.87N
R _U	4475.87N	-

Fuerza	Tensión	Compresión
AB	-	-12071.57N
AC	-	-
R _A	12071.57N	-
BC	10805.61N	-
BD	56777.03N	-
CE	-	-49968.19N
CD	-	-10805.61N
DF	42390.76N	-
DE	9539.65N	-
EF	-	-9539.65N
EG	-	-36477.09N
FH	29794.83N	-
GI	-	-24776.33N
GF	8273.70N	-
GH	-	-8273.70N
HI	7007.74N	-
HJ	18989.22N	-
IK	-	-14034.03
IJ	-	-7007.74N
JL	9973.95N	-
JK	5741.78	-
KM	-	-5913.93N
KL	-	-5741.78N
LN	2749.04N	-
LM	4475.82N	-
MO	415.83N	-
MN	-	-4475.82N
NP	-	-2685.55N
NO	3209.86N	-
OQ	4955.25N	-
OP	-	-3209.86N
PR	-	-6329.8
PQ	1943.90N	-
QS	7704.33N	-
QR	-	-1943.90N
RT	-	-5434.62N
RS	-	-3209.87N
SU	3164.90N	-
ST	3209.87N	-
TU	-	-4475.87N
R _U	4475.87N	-

The vector geometry course shows that there are other methods for obtaining volume and area of a particular figure; for the specific case of this project we used the formulas of dot product and cross product.

For the design of the project the AutoCAD program was used; tools like these allow the previous visualization of all type of construction works and this helps to determine whether it is viable or not, as well as offering other types of options to access the values of the measures. Likewise, this provides the opportunity to get the views in the American and European systems and to change the scales as it suits the user.

The views obtained from the model of the pedestrian bridge and their respective measurements are shown in the Figures 4, and 5.

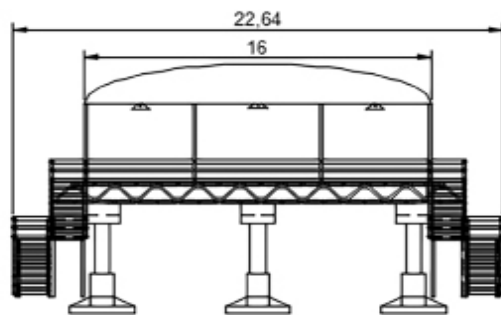


Figure 4. Front view.

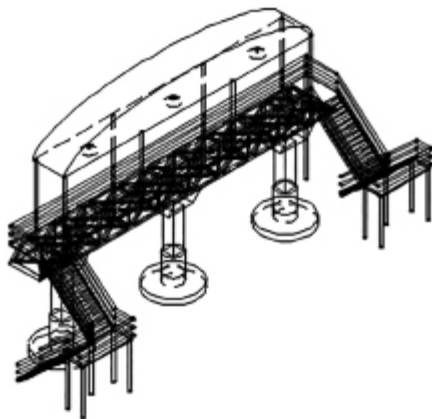


Figure 5. Left side view.

Conclusions

Calculus was an important subject because it allowed to find the values of the points raised in the objectives and also, provided solutions for the area under the curve. It is important to know that the area is an approximation because these process consist in draw rectangles under the function and get their area. Taking into account the calculation of the live and dead loads, can be found out the maximum weight that the bridge can support and what bars in the truss are in compression or tension. Thirdly, the area and the volume have determined the quantity of steel that will be used in the construction of the columns and the floor. Finally, the AutoCAD software was a useful tool in the project because it allowed to have a perspective about the bridge in 3D and 2D plans.

Keeping in mind the results obtained, it can be concluded that is very important to analyze the structure design because the influential factors like the loads, the dimensions or the areas can determine the actual status of the structure and its behavior in the future. Also, with this is possible to know the capacity of the complete structure and its limits.

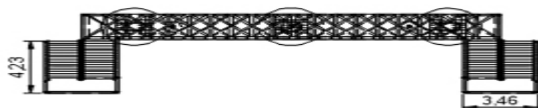


Figure 6. Pedestrian bridge in 3d.

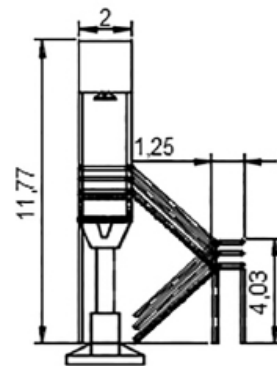


Figure 7. Bottom view.

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